Statistics of S-matrix poles for chaotic systems with broken time reversal invariance: a conjecture

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In the framework of a random matrix description of chaotic quantum scattering the positions of $S-$matrix poles are given by complex eigenvalues Z_i of an effective non-Hermitian random-matrix Hamiltonian. We put forward a conjecture on statistics of Z_i for systems with broken time-reversal invariance and verify that it allows to reproduce statistical characteristics of Wigner time delays known from independent calculations. We analyze the ensuing two-point statistical measures as e.g. spectral form factor and the number variance. In addition we find the density of complex eigenvalues of real asymmetric matrices generalizing the recent result by Efetov [17].

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One of the basic concepts in chaotic quantum scattering is the notion of resonances, representing long-lived intermediate states to which bound states of a "closed" system are converted due to coupling to continua. On a formal level resonances show up as poles of the $M \times M$ scattering matrix $S_{ab}(E)$ occuring at complex energies $\mathcal{E}_k = E_k - (i/2)\Gamma_k$, where E_k is called the position and Γ_k the widths of the corresponding resonance, and M is the number of channels open in a given interval of energies. Recently, advances in computational techniques made available resonance patterns of high accuracy to be obtained for realistic models of atomic and molecular chaotic systems [1].

As is well-known, universal statistical properties of bound states in the regime of quantum chaos have their pattern in statistics of real eigenvalues of large random matrices [2,3]. The methods to adjust random matrix description to the case of resonance scattering in open quantum systems are very well-known since the pioneering work by the Heidelberg group [4]. The method proved to be very fruitful and allowed to calculate different universal characteristics of chaotic scattering, see papers [5,6] for a thorough discussion of recent developments.

In the framework of this approach the S-matrix poles (resonances) are just the complex eigenvalues of an effective random matrix Hamiltonian $\mathcal{H}_{eff} = H - i\Gamma$. Here H is a large self-adjoint $N \times N$ matrix of appropriate symmetry serving to describe the statistical properties of the *closed* counterpart of the scattering system under consideration. The $N\times N$ matrix Γ is to describe a possibility of transitions from the states described by H to the outer world via M open channels and is related to the $N\times M$ matrix W of transition amplitudes as $\Gamma = \pi W W^\dagger$. Such a form is actually dictated by the requirement of $S-$matrix unitarity and ensures that all the $S-$matrix poles are in the lower half-plane of complex energies as required by causality.

In spite of quite substantial analytical [7,8,5] and numerical [9] work on properties of $\mathcal{H}_{eff}$ our actual knowledge of resonance statistics is rather limited. Apart from the simplest perturbative Porter-Thomas treatment [10] (as well as its more advanced variants [11]) the results available on statistics of $S-$ matrix poles are: (i) the joint probability density of all resonances for the system with one open channel and unbroken time-reversal invariance (TRI) [7], (ii) the mean density of S-matrix poles in the complex plane for large number of open channels $M \sim N \gg 1$ [8], (iii) the mean density of S-matrix poles for arbitrary $M \ll N$ for the case of broken TRI [5]. In particular, no information about two-point correlations between different poles is available to our best knowledge.

The situation improves drastically if one replaces the physically motivated matrix Γ introduced above by an (unphysical) $N \times N$ Hermitian matrix A with independent, Gaussian distributed elements and considers H to be taken from the Gaussian Unitary Ensemble (GUE), thus restricting attention to the systems with broken TRI. When the variance of both H and A coincide, such an ensemble was studied long ago by Ginibre [12], who managed to find all the correlation functions of the eigenvalues in the complex plane. They turned out to be quite different from those known for the self-adjoint Gaussian random matrices with real eigenvalues, studied by Wigner,Dyson,Mehta and others [2].

At the same time it is clear that reducing the variance of A as compared to that of H drives the ensemble towards GUE. The existence of a nontrivial regime of *weak non-Hermiticity* was recognized in our preceding works [13,14]. This regime occurs when $\mathrm{Tr}A^2 \sim \frac{1}{N} TrH^2$ in the limit of large N. Under this condition the imaginary part Y_k of a typical complex eigenvalue $Z_k = x_k - iY_k$ is comparable with the mean *separation* $\Delta = \langle x_k - x_{k+1} \rangle \sim \frac{1}{N}$ between neighboring eigenvalues along the real axis. Exploiting the method of orthogonal polynomials we demonstrated [14] that all statistical properties of $\mathcal{H} = H - iA$ in this regime can be described in terms of a kernel $K(Z_1, Z_2)$ depending on two com-

plex coordinates $Z_{1,2}=x\pm\omega/2N-iy_{1,2}/N$. In particular, the mean eigenvalue density $\rho(Z)=\langle\sum_k\delta^{(2)}(Z-Z_k)\rangle$ is given by $\rho(Z)=K(Z,Z)$ and the two-point *cluster function* $\mathcal{Y}_2(Z_1,Z_2)$ defined via the relation:

$$\langle\rho(Z_1)\rho(Z_2)\rangle_c=\langle\rho(Z)\rangle\,\delta^{(2)}(Z_1-Z_2)-\mathcal{Y}_2(Z_1,Z_2) \quad (1)$$

is given by $\mathcal{Y}_2(Z_1,Z_2)=|K(Z_1,Z_2)|^2$.

Unfortunately, such a detailed information is of no direct use for the physically motivated case of chaotic scattering. Nevertheless, the insights provided by the Gaussian case combined with the existent knowledge of the resonance statistics allowed us to put forward a well-grounded conjecture about statistics of complex eigenvalues of weakly non-Hermitian ensemble of the type $H-i\Gamma$ for *any* given Hermitian matrix Γ. This issue is the content of the main part of the present paper.

Before formulating the conjecture, let us recall that the GUE ensemble is an *invariant* one, i.e. its statistics is the same independently on the basis chosen. Therefore, one always can go to the eigenbasis of the matrix $\hat{\Gamma}$ and consider it to be diagonal with eigenvalues $\gamma_i,\ i=1,...,N$. In what follows we find it convenient to characterize the matrix $\hat{\Gamma}$ by the following function:

$$f_\Gamma(z)=\sum_i\ln\left(1+\frac{z}{\pi\nu(x)g_i}\right) \quad (2)$$

where $g_i=\frac{1}{2\pi\nu(x)}(\gamma_i+\gamma_i^{-1})$ and $\nu(x)=\frac{1}{2\pi}\sqrt{4-x^2}$ stands for the Wigner semicircular density of real eigenvalues of the Hermitian part $\hat{H}$ of the matrices $\mathcal{H}$.

Now we put forward the following **conjecture**: Suppose the function $f_\Gamma(z)$ defined above has a finite limit when $N\to\infty$. Then the statistics of eigenvalues Z_i of the corresponding almost Hermitian ensemble $\mathcal{H}_{eff}$ in the limit $N\gg 1$ is completely determined by the kernel

$$K(Z_1,Z_2)=\frac{N^2}{4\pi^2}e^{i\frac{x}{2}(y_2-y_1)}\int_{-\pi\nu_{sc}(x)}^{\pi\nu_{sc}(x)}du\,e^{-u(y_1+y_2)+iu\omega+f_\Gamma(u)} \quad (3)$$
$$\times\left(\int_{-\infty}^{\infty}dk_1e^{-ik_1y_1-f_\Gamma(-ik_1/2)}\int_{-\infty}^{\infty}dk_2e^{-ik_2y_2-f_\Gamma(-ik_2/2)}\right)^{1/2}$$

In particular, the mean eigenvalue density $\rho(Z)=\langle\sum_k\delta^{(2)}(Z-Z_k)\rangle$ is given by $\rho(Z)=K(Z,Z)$ and the two-point cluster function is $\mathcal{Y}_2(Z_1,Z_2)=|K(Z_1,Z_2)|^2$.

Let us now systematically veryfy the compatibility of our conjecture with the known properties of almost-Hermitian matrices of various types.

The simplest test is to make sure that for a Gaussian $\hat{\Gamma}$ such that $Tr\Gamma^2\sim\frac{1}{N}TrH^2$ we are back to results proved in [14]. Indeed, for this case a typical eigenvalue $\gamma_i\sim N^{-1/2}\ll 1$, hence $g_i^{-1}\approx 2\pi\nu(x)\gamma_i\ll 1$ and in the limit of large N one can expand $f(z)$ in a series. The first term (proportional to z) vanishes in the limit $N\to\infty$ because of the symmetry of the distribution of eigenvalues of $\hat{\Gamma}$ around zero. Thus, the leading term turns out to be proportional to z^2. The corresponding Gaussian integrals over $k_{1,2}$ in Eq.(3) can be performed exactly, the resulting kernel reproducing that found in [14].

Let us now consider the mean eigenvalue density $\rho(Z)=K(Z,Z)$. The "physical" case $\Gamma=\pi WW^\dagger$ with a finite number M of open channels (i.e, with a finite number M of *positive* non-zero eigenvalues $\gamma_i,\ i=1,..,M$) was considered earlier in [5]. The result coincides with that following from Eq.(3).

Actually, one can easily adopt the methods used in [5] to satisfy oneself that the validity of the corresponding expression is not restricted by the case of positive γ_i, but rather extends to an arbitrary set of eigenvalues. This fact provides a proof of our conjecture on the level of the mean eigenvalue density.

Let us now show that our conjecture survives a much more stringent test on the level of two-point correlations. For this purpose let us invoke the notion of the so-called (energy dependent) *Wigner time delay* defined in terms of resonance positions E_k and widths Γ_k as (see [5] for more details):

$$\tau_w(E)=\frac{1}{M}\sum_k\frac{\Gamma_k}{(E-E_k)^2+\Gamma_k^2/4} \quad (4)$$

Using this expression it is easy to relate the correlation function $\langle\tau_W(E_1)\tau_W(E_2)\rangle_c$ of the Wigner time delays at two different energies $E_{1,2}=E\pm\Omega/2$ to the two-point correlation function $\langle\rho(Z_1)\rho(Z_2)\rangle_c$ of the densities of $S-$matrix poles in the complex plane $Z=x-iY$. Considering the energy difference $E_1-E_2=\Omega$ to be comparable with the mean level spacing $\Delta=1/\nu(E)N$ and exploiting the fact that both the mean density $\langle\rho(x,Y)\rangle$ and the cluster function $\mathcal{Y}_2(x_1,x_2,Y_1,Y_2)$ change with $x=(x_1+x_2)/2$ on a scale much larger than Δ, one can perform the $x-$ integration explicitly. After this it is convenient to pass to the scaled variables:

$$\tilde{\tau_W}=\frac{M\Delta}{2\pi}\tau_w;\ \tilde{\omega}=\frac{\pi\Omega}{\Delta};\ y=\frac{\pi Y}{\Delta};\ \omega_x=\frac{\pi\omega}{\Delta},$$

with $\omega=x_1-x_2$, to rescale the cluster function and the density as follows:

$$\tilde{\rho}_E(y)=\frac{\Delta^2}{\pi}\langle\rho(E,Y)\rangle;\tilde{\mathcal{Y}}_2(E,\omega_x,y_1,y_2)=\frac{\Delta^4}{\pi^2}\mathcal{Y}_2(E,\omega,Y_1,Y_2)$$

and to make a Fourier transformation with respect to $\tilde{\omega}$. These manipulations allowed us to write down the relation we were looking for in quite a compact form:

$$\frac{1}{\pi}\int_{-\infty}^{\infty}d\tilde{\omega}e^{i\tilde{\omega}t}\left\langle\tilde{\tau}_W(E+\frac{\Delta}{\pi}\tilde{\omega})\tilde{\tau}_W(E-\frac{\Delta}{\pi}\tilde{\omega})\right\rangle_c=\int_0^{\infty}dye^{-2ty}\tilde{\rho}_E(y)$$
$$-\int_{-\infty}^{\infty}d\omega_x\int_0^{\infty}dy_1\int_0^{\infty}dy_2e^{-t(y_1+y_2+i\omega_x)}\tilde{\mathcal{Y}}_2(E,\omega_x,y_1,y_2) \quad (5)$$

Such a relation between the correlation function of time delays and two-point cluster function provides a possibility of the most non-trivial test of our conjecture. Indeed, both the left-hand side and the first term in the right-hand side are known from independent calculations [5], which allows us to rewrite the Eq.(5) for $t>0$ as:

$$
\begin{aligned}
&-\int_{-\infty}^{\infty} d\omega_x \int_0^{\infty} dy_1 \int_0^{\infty} dy_2 e^{-t(y_1+y_2+i\omega_x)} \tilde{\mathcal{Y}}_2(E,\omega_x,y_1,y_2) \\
&= \frac{1}{2}\theta(2-t)\int_{-min(t,1)}^{1-t} d\lambda \prod_{i=1}^{M} \frac{g_i+\lambda}{g_i+\lambda+t} \qquad (6)
\end{aligned}
$$

Although this relation does not provide a possibility to extract the cluster function in a unique way, it is easy to satisfy oneself that a direct subtitution of $\mathcal{Y}_2(Z_1,Z_2)=|K(Z_1,Z_2)|^2$, with the kernel taken from Eq.(3) into the left-hand side of Eq.(6) produces after rescaling and integration exactly the right-hand side of Eq.(6). This fact provides the strongest support to our conjecture.

Having at our disposal the conjectured form of the cluster function $\mathcal{Y}(Z_1,Z_2)$ it is interesting to calculate other related quantities frequently used in applications, such as the spectral formfactor and the number variance. For the Gaussian case such a calculation was performed in [14]. For the physically interesting case of resonance scattering the formfactor can be obtained along the same lines and is equal to:

$$
\begin{aligned}
b(q_1,q_2,k) &= \int_{-\infty}^{\infty} d\omega_x \int_0^{\infty} dy_1 \int_0^{\infty} dy_2 e^{2\pi i(q_1y_1+q_2y_2+k\omega)} \mathcal{Y}_2(Z_1,Z_2) \\
&= \frac{N^4}{2}\theta(\nu-|k|)\int_{-(\nu-|k|)}^{(\nu-|k|)} dv \prod_{i=1}^{M} \frac{(g_i\nu+v)^2-k^2}{(g_i\nu+v-iq_1)(g_i\nu+v-iq_2)} \qquad (7)
\end{aligned}
$$

with $\nu\equiv\nu(X)$. The variance of the number of resonances in a strip $0<\mathrm{Re}Z<L;\ -\infty<\mathrm{Im}Z<\infty$ of the width $L=L_x\Delta$ comparable with Δ can be expressed in terms of $b(0,0,k)$, see [14]. In particular, for the case of equivalent scattering channels: $g_i=g,\ i=1,...,M$ it is given by:

$$
\begin{aligned}
\Sigma_2(L_x) = L_x - \frac{1}{\pi^2}\int_0^1 dk k^{-2}\sin^2(\pi k L_x) \\
\times \int_{-(1-k)}^{(1-k)} dv \left[1-k^2(g+v)^{-2}\right]^M \qquad (8)
\end{aligned}
$$

where we have used $\pi\nu(0)=1$. Let us discuss for simplicity its typical features for the symplest case of only one open channel $M=1$. We are actually interested in deviations of the number variance from its value known for Hermitian GUE matrices. One finds:

$$
\begin{aligned}
\delta\Sigma(L_x) &= \Sigma_2(L_x) - \Sigma_2^{(GUE)}(L_x) \qquad (9) \\
&= \frac{2}{\pi^2}\int_0^1 dk \sin^2(\pi k L_x)\frac{(1-k)}{g^2-(1-k)^2}
\end{aligned}
$$

Usually, one is interested in the behaviour of the number variance for $L_x \gg 1$. Then, for any $g>1$ one finds that the difference $\delta\Sigma(L_x)$ tends to a constant value $\frac{1}{2\pi^2}\ln[g^2/(g^2-1)]$. The so-called "perfect coupling" case $g=1$ is known for being specific: e.g. the resonance density shows at $g=1$ the powerlaw behaviour [5]. On the level of number variance such a specificity is reflected in a logarithmically growing difference: $\delta\Sigma(L_x)\propto\ln L_x$.

Finally, the knowledge of the cluster function allows one to find the small-distance behaviour of the so-called nearest-neighbour distance distribution $p(Z_0,S\ll\Delta)$, see e.g. [14]. As expected [9], the leading term for $S\to 0$ turns out to be always cubic: $p(Z_0,S)\sim S^3$, as long as the system is open: $g\sim 1$. However, it is interesting to note that for very weakly open systems: $g\gg 1$ the cubic law is modified in such a way, that in a parametrically large region $g^{-1}\ll S/\Delta<\mathrm{Im}Z_0/\Delta$ the behavour becomes $p(Z_0,S)\sim S^{5/2}$ (cf. [14]). Unfortunately, the density of complex eigenvalues is exponentially small under the condition $\mathrm{Im}Z_0/\Delta\gg g^{-1}$, the fact hindering a possibility to observe such an unusual regime numerically.

Let us now turn our attention to another class of random matrices with complex eigenvalues which attracted much attention recently. Namely, we consider the ensemble of weakly asymmetric matrices with real elements. Such matrices can always be presented in the form $H+A$, with H being real symmetric (hence, taken from GOE) and A being a real antisymmetric: $A_{ij}=-A_{ji}$ such that $N\mathrm{Tr}A^2\sim\mathrm{Tr}H^2$ in the limit of large N. The case of matrices A with independent, identically distributed Gaussian entries was studied by various authors and by different methods [17,16,15]. In particular, the following unusual property was detected numerically in [16] and proved analytically in [15,17]: the *finite fraction* of eigenvalues stays on the real axis even for $A\neq 0$. This fact should be contrasted with the corresponding property of earlier discussed weakly non-Hermitian matrices whose eigenvalues with probability unity have a finite imaginary part as long as $\hat{\Gamma}\neq 0$. More recently, the interest to the ensemble of slightly asymmetric real matrices arose after the work by Efetov [17], who discovered its relation to an interesting problem of motion of vortices in disordered superconductors with columnar defects [18]. Efetov calculated explicitly the mean density of eigenvalues for the Gaussian A. Shortly after Halasz et al [19] discovered that Efetov's result describes also the density of real eigenvalues of some matrices appearing in a random matrix approach to the problem of spontaneous breaking of chiral symmetry in QCD. An interesting feature was that the perturbation considered in [19] forcing the eigenvalues to leave the real line was not at all random. Translating these results to the ensemble of random real non-symmetric matrices it is natural to expect that for antisymmetric perturbations of the type $A=\begin{pmatrix} 0 & \mu\mathbf{1} \\ -\mu\mathbf{1} & 0 \end{pmatrix}$, with a constant μ being of the order

of $1/\sqrt{N}$ Efetov's formula should still provide the correct eigenvalue density.

This fact motivated us to reconsider the problem of calculation of the mean eigenvalue density for a general fixed real antisymmetric matrix A as it is done above for the case of almost-Hermitian matrices. Invoking again the arguments of the rotational invariance, it is enough to consider the matrices A of the following structure: $A = \mathrm{diag}(A_1, ..., A_N)$, with each block A_i being 2×2 matrix of the form $A_i = \begin{pmatrix} 0 & \mu_i \\ -\mu_i & 0 \end{pmatrix}$, since an arbitrary antisymmetric A can be reduced to such a form by an orthogonal rotation. The density of complex eigenvalues for the matrix $H+A$ can be found by a straightforward modification of the calculation presented in [17]. Introducing the scaled variable $y = \pi\nu(X)N\mathrm{Im}Z$ and rescaling the density correspondingly: $\rho_X(y) = \langle\rho(Z)\rangle/(N\pi\nu(X))^2$, one finds:

$$\rho_x(y) = \delta(y)\int_0^1 du e^{\frac{1}{2}[f_A(\pi\nu u)+f_A(-\pi\nu u)]} + \frac{1}{2\pi}\int_0^1 du u \sinh(|y|u) e^{\frac{1}{2}[f_A(\pi\nu u)+f_A(-\pi\nu u)]} \times \int_{|y|}^{\infty} ds \int_{-\infty}^{\infty} dk e^{-iks-\frac{1}{2}[f_A(i\pi\nu k)+f_A(-i\pi\nu k)]} \tag{10}$$

where the function $f_A(z)$ is given again by Eq.(2) with γ_i replaced by μ_i.

From the derived expression one immediately infers that if a typical μ_i is of the order of $N^{-1/2}$, the function $f_A(u)$ can be expanded up to a first nonvanishing order such that $f_A(u) + f_A(-u) \propto \mathrm{Tr}A^2u^2$ and the corresponding expression coincides with that found by Efetov. As such, Efetov's formula is indeed applicable also for constant matrices A of the type described above.

We see that the most striking qualitative features of the Efetov's formula: i)a nonvanishing density of real eigenvalues and ii) a linear increase with $|y|$ of the probability density to have a finite imaginary part -persist for any antisymmetric perturbation A. The strongest quantitative deviation from Efetov's result occurs in the case of finite-rank perturbations A such that $\mu_i = 0$ for $i > M$. In particular, one can show that if at least one of the quantities $g_i = \frac{1}{2\pi\nu}(\mu_i + \mu_i^{-1})$. is equal to unity, the mean density decays asymptotically as $\rho(y \gg 1) \propto y^{-2}$. Such a slow power law decay should be contrasted with the Gaussian case when one always has a very sharp cutoff of the density for large enough y. For the general case of a finite-rank antisymmetric perturbation such that $g_i \neq 1$ the density is cut exponentially at $y \sim (g_i - 1)^{-1}$.

In conclusion, we put forward a conjecture on statistics of S-matrix poles Z_i for systems with broken time-reversal invariance and verified that it is perfectly compatible with the existent knowledge on quantum chaotic scattering. In particular, our conjecture allowed us to reproduce statistical characteristics of Wigner time delays known from independent calculations. We analyzed the ensuing two-point statistical measures as e.g. spectral form factor, number variance and small distance behavior of the nearest neighbor distance distribution $p(Z_0, S)$. In the final part of the paper we calculated the density of complex eigenvalues of an ensemble of real weakly asymmetric matrices. The expression obtained generalizes the recent result by Efetov [17] to the case of an arbitrary antisymmetric perturbation.

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